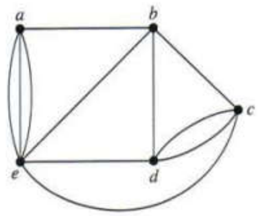
Discrete Mathematics

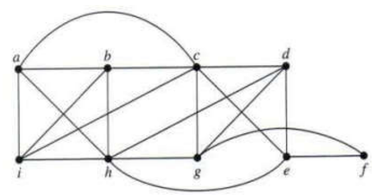
<u>Tutorial 12 - Answer</u>

<u>Refer to Chapter 9.5, 9.7, and 9.8</u>

1. Determine whether the following graphs have an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



a)



b)

Answer

a) All the vertex degrees are even
Euler circuit: a, b, c, d, c, e, d, b, e, a, e, a

b) All the vertex degrees are even
Euler circuit. One such circuit is a, i, h, g, d, e, f, g, c, e, h, d, c, a, b, i, c, b, h, a

2. For which values of *n* do these graphs have an Euler circuit?
   a) K_n b) C_n c) W_n

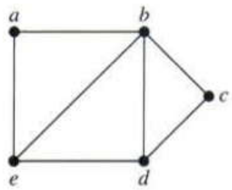
Answer:

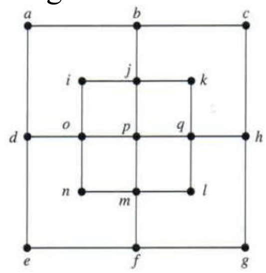
a) The degree of the vertices (n-1) are even if and only if n is odd. Therefore there is an Euler circuit if and only if n is odd (and greater than 1, of course).

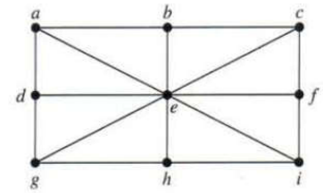
b) For all $n \geq 3$, clearly C_n has an Euler circuit.

c) Since the degrees of the vertices around the rim are all odd, no wheel has an Euler circuit.

3. Determine whether the following graphs have a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

a) 

b) 

c) 

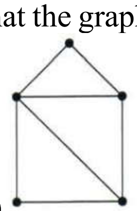
Answer:

a) a, b, c, d, e, a.

b) This graph has no Hamilton circuit. If it did, then the circuit would have to contain edges {d, a}and {a, b}, since these are the only edges incident to vertex a. By the same reasoning, the circuit would have to contain the other six edges around the outside of the figure. These eight edges already complete a circuit, and this circuit omits the nine vertices on the inside. Therefore there is no Hamilton circuit.

c) a, d, g, h, i, f, c, e, b, a.

4. For each of these graphs, determine (i) whether Dirac's Theorem can be used to show that the graph has a Hamilton circuit, (ii) whether Ore's Theorem can be used to show that the graph has a Hamilton circuit, and (iii) whether the graph has a Hamilton circuit.

a) 

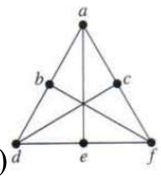
b) 

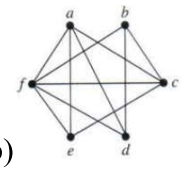
Answer:

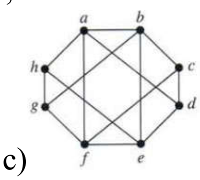
a) (i) No. (ii) No. (iii) Yes.

b) (i) Yes. (ii) Yes. (iii) Yes.

5. Determine whether the given graphs are planar. If so, draw it so that no edges cross.

a) 

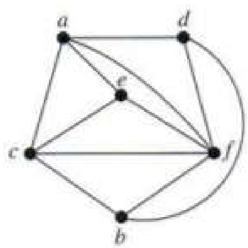
b) 

c) 

Answer:

a) No. There is in fact an actual copy of $K_{3,3}$, with vertices a, d, and f in one set and b, c, and e in the other.

b) Yes.



c) No. The graph is a supergraph of $K_{3,3}$ with vertices a, c, and e in one set and b, d, and f in the other.

6. Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?

Answer:

$r = e - v + 2$. Therefore $v = e - r + 2 = 30 - 20 + 2 = 12$.

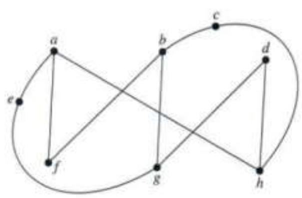
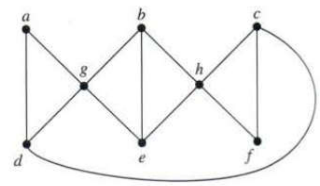
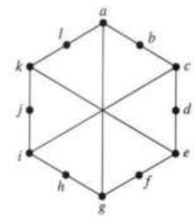
7. Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?

a) K_5 b) K_6 c) $K_{3,3}$ d) $K_{3,4}$

Answer:

Only (a) and (c).

8. Determine whether the given graphs are homeomorphic to K3,3.

a)  b)  c) 

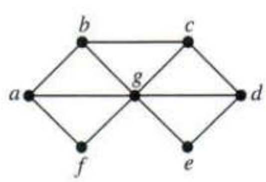
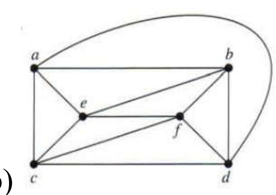
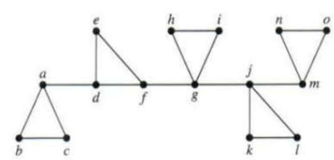
Answer:

a) No. By rerouting the edge between a and h we see that it is planar.

b) No. This graph is planar.

c) Yes. Replace each vertex of degree two and its incident edges by a single edge. Then the result is K3,3: the parts are {a, e, i} and {c, g, k}.

9. Find the chromatic number of the given graphs.

a)  b)  c) 

d) K_4 e) $K_{3,4}$ f) $K_{4,5}$

Answer:

a) At least 3 colors are needed: one for vertices a, c, and e, one for b, d, and f, and one for g.

b) At least 3 colors are needed: one for vertices b and c, one for a and f, and one for d and e.

c) At least 3 colors are needed: one for vertices a, e, h, k and o, one for b, d, g, l and m, and one for c, f, i, j and n.

d) 4

e) 2

f) 2