Discrete Mathematics I
Tutorial 07 - Answer

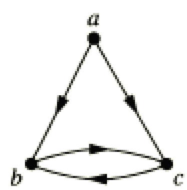
Refer to Chapter 5.1, 5.2, 5.3

1. List the ordered pairs in the relation R from A = {0,1,2,3,4} to B = {0,1,2,3}, where (a,b) $\in$R if and only if
   a) a = b   b) a + b = 4   c) a>b

Answer:
   a) {(0,0), (1,1), (2,2), (3,3)}
   b) {(1,3), (2,2), (3,1), (4,0)}
   c) {(1,0), (2, 0), (3,0), (4,0), (2,1), (3,1), (4,1), (3,2), (4,2), (4,3)}

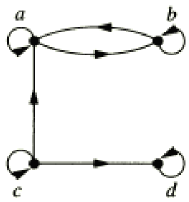
2. Draw the directed graph and matrix for the following relation:
   a) {(a,b), (a,c), (b,c), (c,b)} on the set A = {a, b, c}
   b) {(a, a), (a, b), (b, a), (b, b), (c, c), (c, a), (c, d), (d, d)} on the set A = {a, b, c, d}

Answer:
   a)



$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   b)



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find the matrices that represent

a) R_2^{-1} b) $\overline{R_1}$ c) $R_1 \cup R_2$

d) $R_1 \oplus R_2$ e) $R_2 \circ R_1$ f) R_1^2

Answer:

a) Matrix for $R_2^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

b) Matrix for $\overline{R_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

c) Matrix for $R_1 \cup R_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

d) Matrix for $R_1 \oplus R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

e) Matrix for $R_2 \circ R_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

f) Matrix for $R_1^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

4. How many relations are there on the following set:
   a) a set 4 elements {a, b, c, d}
   b) a symmetric relation R on the set with n elements?
   c) a asymmetric relation R on the set with n elements
   d) a irreflexive relation R on the set with n elements

Answer:

a) $2^{4^2} = 65535$
b) $2^{n(n+1)/2}$
c) $3^{n(n-1)/2}$
d) $2^{n(n-1)}$

5. Let R be the relation on the set {1,2,3,4,5} containing the ordered pairs (1,1), (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,2), (4,5), (5,1), (5,2), and (5,4). Find
   a) R^2 b) R^3 c) R^5

Answer:

a) R^2={(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)}

b) c)
$R^3 = R^5$

$$R = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$R^2 = R \circ R = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$R^3 = R^2 \circ R = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$R^5 = R^4 = R^3$$

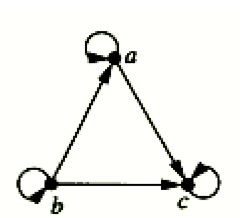
6. Determine whether the following relation ***R*** whether it is symmetric, antisymmetric, asymmetric, transitive, irreflexive and reflexive.
   a) {(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)} on set {1,2,3,4}
   b) {(2,4), (4,2)} on set {1,2,3,4}
   c) {(1,1), (2,2), (3,3), (4,4)} on set {1,2,3,4}
   d) {(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)} on set {1,2,3,4}
   e) "There are no common links found on both Web page a and Web page b" on the set of all web pages
   f) "There is a Web page that includes link to both Web page a and Web page b." on the set of all web pages
   g) $xy \geq 0$ on the set of all real numbers
   h) $x = \pm y$ on the set of all real numbers
   i) $x=1$ or $y=1$ on the set of all real numbers
   j) $x-y$ is a rational number on the set of all real numbers
   k)

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

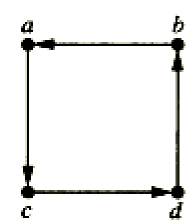
   l)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

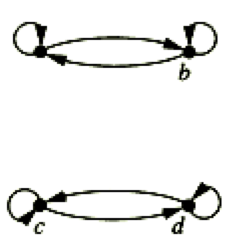
   m)



   n)



   o)



Answer:

a) transitive
b) symmetric, irreflexive
c) reflexive, symmetric, antisymmetric, transitive
d) irreflexive
e) Symmetric, irreflexive
f) Symmetric, reflexive
g) symmetric,transitive, reflexive
h) symmetric, transitive
i) symmetric
j) symmetric, transitive, reflexive
k) Reflexive, antisymmetric
l) Symmetric, irreflexive
m) Reflexive, Antisymmetric, transitive
n) asymmetric,
o) reflexive, symmetric, transitive

7) Suppose that R and S are reflexive relation on a set A prove or disprove each of these statements

a) $R \cup S$ is reflexive
b) $R - S$ is irreflexive
c) $R \oplus S$ is irreflexive
d) $S \circ R$ is reflexive

Answer:

Given Reflexive
$\forall a \in A \ (a,a) \in R$, $\forall b \in A \ (b,b) \in S$

a) True
Proof:
Since $\forall a \in A \ (a,a) \in R$, $\forall b \in A \ (b,b) \in S$
$\forall a \in A \ (a,a) \in R \bigcup \mathrm{S}$
So $R \bigcup S$ are also reflexive

b) True
Proof:
$\forall a \in A \ (a,a) \in R, \ (a,a) \in \mathrm{S}$
$(a,a) \notin R \text{-} \mathrm{S}$
So $R - S$ is irreflexive

c) True
Proof:

$R \oplus S = R \bigcup S - R \bigcap S$

As $(a,a) \in R \bigcup \mathrm{S}$, which is shown in (a). Similarly, $(a,a) \in R \bigcap \mathrm{S}$

Therefore, $(a,a) \notin R \bigcup \mathrm{S} - R \bigcap \mathrm{S}$

So $R \oplus S$ is irreflexive

d) True

Proof:

$\forall a,b \in A,\ (a,b) \in S \circ R$

It means there is a element z, where $(a,z) \in R$ and $(z,b) \in S$

As both S and R are reflexive, for each element a in A, $(a,a) \in R$ and $(a,a) \in S$. Therefore, $S \circ R$ is reflexive