Discrete Mathematics I

Tutorial 04 - Answer

Refer to Chapter 2.1 and 2.2

1. List the members of the set $\{x \mid x$ is an integer such that $x^2 = 2\}$.

Answer:

$\varnothing$

2. Use set builder notation to give a description of each of these sets.
   a) $\{0, 3, 6, 9, 12\}$
   b) $\{-3, -2, -1, 0, 1, 2, 3\}$

Answer:

a) $\{x \mid x = 3n$, where n is 0, 1, 2, 3, 4 $\}$
b) $\{x \mid x$ is a positive integer and $|x| \leq 3\}$

3. Determine whether each of these pairs of sets are equal.
   a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$, $\{5, 3, 1\}$
   b) $\{\{1\}\}$, $\{1, \{1\}\}$
   c) $\varnothing$, $\{\varnothing\}$

Answer:

(a) Yes, for they have the same element
(b) No, $\{1\}$ is a set, so $\{1,\{1\}\}$ has two elements, while $\{\{1\}\}$ has only one
(c) No, $\varnothing$ is a element but $\{\varnothing\}$ is a set.

4. For each of the following sets, determine whether 2 or $\{2\}$ is an element of that set.
   a) $\{x \in \boldsymbol{R} \mid x$ is an integer greater than 1$\}$
   b) $\{x \in \boldsymbol{R} \mid x$ is the square of an integer$\}$
   c) $\{2,\{2\}\}$
   d) $\{\{2\},\{\{2\}\}\}$
   e) $\{\{2\},\{2,\{2\}\}\}$
   f) $\{\{\{2\}\}\}$

Answer:

	2	{2}
a)	Yes	No
b)	No	No
c)	Yes	Yes
d)	No	Yes
e)	No	Yes
f)	No	No

5. Determine whether these statements are true or false.
   a) $\varnothing \in \{\varnothing\}$
   b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
   c) $\{\varnothing\} \in \{\varnothing\}$
   d) $\{\varnothing\} \in \{\{\varnothing\}\}$
   e) $\{\varnothing\} \subset \{\varnothing, \{\varnothing\}\}$
   f) $\{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$
   g) $\{\{\varnothing\}\} \subset \{\{\varnothing\}, \{\varnothing\}\}$

Answer:

(a) Yes.
(b) Yes.
(c) No.
(d) Yes.
(e) Yes.
(f) Yes.
(g) No

6. Let A = {a, b, c, d} and B = {y, z}. Find
   a) A x B
   b) B x A

Answer:

a) A x B = {(a,y),(a,z),(b,y),(b,z),(c,y),(c,z),(d,y),(d,z)}
b) B x A = {(y,a),(y,b),(y,c),(y,d),(z,a),(z,b),(z,c),(z,d)}

7. Explain why A x B x C and (A x B) x C are not the same.

Answer:

Assume (a,b,c) ∈ A x B x C, then (a,b,c) is 3-tuple.
Assume ((a,b),c) ∈ (A x B) x C, then ((a,b),c) is 2-tuple.
Therefore, they are different.

8. Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find
   a) $A \cup B$
   b) $A \cap B$
   c) $A - B$
   d) $B - A$
   f) $A \oplus B$

Answer:

   a) $A \cup B = \{a,b,c,d,e,f,g,h\}$
   b) $A \cap B = \{a,b,c,d,e\}$
   c) $A-B = \varnothing$
   d) $B-A = \{f,g,h\}$
   f) $A \oplus B = \{f,g,h\}$

9. Let A, B, and C be sets. Show that
   a) $(A \cap B) \subseteq A$.
   b) $A \subseteq (A \cup B)$.
   c) $A - B \subseteq A$.
   d) $A \cap (B - A) = \varnothing$.
   e) $(A \cup B) \subseteq (A \cup B \cup C)$
   f) $(A - B) - C = (A - C) - (B - C)$
   g) $(A - C) \cap (C - B) = \varnothing$
   h) $(B - A) \cup (C - A) = (B \cup C) - A$

Answer:

   a) $(A \cap B) \subseteq A$
      Let $x \in (A \cap B)$
      $x \in A$ and $x \in B$
      Therefore, $x \in A$

   b) $A \subseteq (A \cup B)$
      Let $x \in A$
      Since $x \in A$, $x \in A$ or $x \in B$ must be true
      $x \in A$ or $x \in B$ means $x \in (A \cup B)$

   c) $A-B \subseteq A$
      Let $x \in A-B$
      $x \in A$ and $x \notin B$
      Therefore, $x \in A$

   d) $A \cap (B - A) = \varnothing$

$$
\begin{aligned}
& A \cap (B - A) \\
= \; & \{ x \mid (x \in A) \wedge (x \in B \wedge x \notin A)\} \\
= \; & \{ x \mid (x \in A \wedge x \in B \wedge x \notin A)\} \\
= \; & \{ x \mid (\varnothing \wedge x \in B)\} \\
= \; & \varnothing
\end{aligned}
$$

e) $(A \cup B) \subseteq (A \cup B \cup C)$

Let $y \in A \cup B$
$y \in A$ or $y \in B$
Therefore $y \in A$ or $y \in B$ or $y \in C$

f) $(A-B)-C = (A-C)-(B-C)$

$(A-B) - C$
$= (A \cap \overline{B}) - C$
$= (A \cap \overline{B}) \cap \overline{C}$
$= A \cap \overline{B} \cap \overline{C}$

$(A-C)-(B-C)$
$= (A \cap \overline{C})-(B \cap \overline{C})$
$= (A \cap \overline{C}) \cap \overline{(B \cap \overline{C})}$
$= (A \cap \overline{C}) \cap (\overline{B} \cup C)$
$= (A \cap \overline{C} \cap \overline{B}) \cup (A \cap \overline{C} \cap C)$
$= (A \cap \overline{C} \cap \overline{B}) \cup \varnothing$
$= A \cap \overline{C} \cap \overline{B}$

So $(A-B)-C= (A-C)-(B-C)$

g) $(A - C) \cap (C - B) = \varnothing$
Let $x \in (A-C) \cap (C-B)$
$x \in (A \cap \overline{C}) \cap (C \cap \overline{B})$
$x \in A \cap \overline{C} \cap C \cap \overline{B}$
$x \in \varnothing$

h) $(B - A) \cup (C - A) = (B \cup C) - A$.

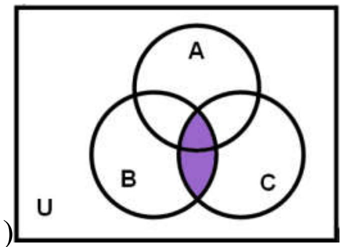
$(B \cup C) - A$
$= (B \cup C) \cap \overline{A}$
$= (B \cap \overline{A}) \cup (C \cap \overline{A})$
$= (B - A) \cup (C - A)$

10. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
   a) $A \cap (B - \overline{C})$
   b) $(A \cap \overline{B}) \cup (A \cap \overline{C})$
   c) $A \subset B$ and $A \subset C$

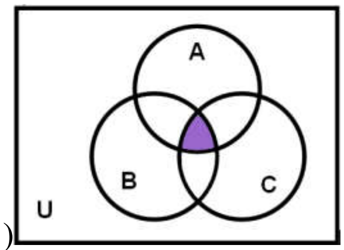
Answer:

a)

$(B - \overline{C})$

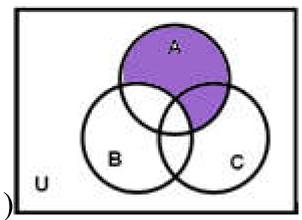


$A \cap (B - \overline{C})$

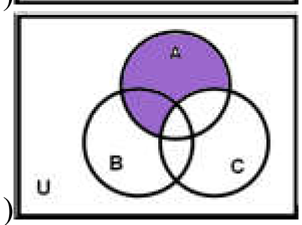


b)

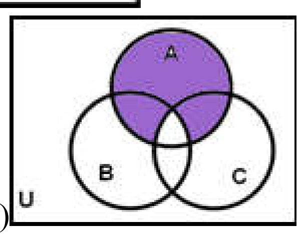
$(A \cap \overline{B})$



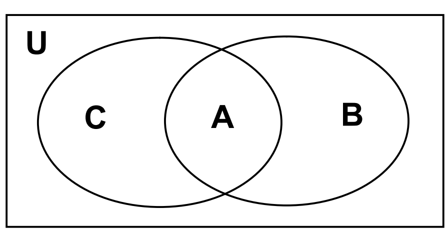
$(A \cap \overline{C})$

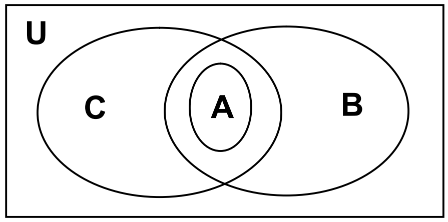


$(A \cap \overline{B}) \cup (A \cap \overline{C})$



c)





← this one is better

11. What can you say about the sets A and B if we know that
    a) $A \cup B = A$?
    b) $A \cap B = A$?
    c) $A - B = A$?
    d) $A \cap B = B \cap A$?
    e) $A - B = B - A$?

Answer:

a) $B \subseteq A$
b) $A \subseteq B$
c) $A \cap B = \varnothing$
d) A and B can be any set
e) $A = B$

12. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,
    a) $A_i = \{i, i+1, i+2, \ldots\}$.
    b) $A_i = \{0, i\}$.
    c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.
    d) $A_i = (i, \infty)$ that is, the set of real numbers x with $x > i$.

Answer:

a) $\bigcup_{i=1}^{\infty} A_i = N^+$

$\bigcap_{i=1}^{\infty} A_i = \phi$

b) $\bigcup_{i=1}^{\infty} A_i = \{x \mid x \in N^+ \ or \ x = 0\}$

$\bigcap_{i=1}^{\infty} A_i = \{0\}$

c) $\bigcup_{i=1}^{\infty} A_i = \{x \in R \mid x > 0\}$

$\bigcap_{i=1}^{\infty} A_i = \{x \in R \mid x \in (0,1)\}$

d) $\bigcup_{i=1}^{\infty} A_i = \{x \in R \mid x > 1\}$

$\bigcap_{i=1}^{\infty} A_i = \phi$

13. Suppose that the universal set is U = {1,2,3,4,5,6,7,8,9,10}.Express each of these sets with bit strings where the i^{th} bit in the string is 1 if i is in the set and 0 otherwise.
   a) {3, 4, 5}
   b) {1, 3, 6, 10}
   c) {2, 3, 4, 7, 8, 9}

Answer:
   a) 00111 00000
   b) 10100 10001
   c) 01110 01110