Discrete Mathematics I
Tutorial 02 - Answer

Refer to Chapter 1.3 and 1.4

1. Let Q(x) be the statement "x + 1 > 2x." If the domain consists of all integers, what are these truth values?
   a) $Q(0)$
   b) $Q(-1)$
   c) $\exists x\, Q(x)$
   d) $\forall x\, Q(x)$
   e) $\exists x\, \neg Q(x)$
   f) $\forall x\, \neg Q(x)$

Answer:

a) True (Q(0) : 0<1)
b) True (Q(-1) : 0>-2)
c) True (e.g. Q(0))
d) False (e.g. Q(1))
e) True ((e) = ¬(e))
f) False ((f) = ¬(c))

2. Determine the truth value of each of these statements if the domain consists of all integer numbers.
   a) $\exists x\, (x^3 = -1)$
   b) $\exists x\, (x^4 < x^2)$
   c) $\forall x\, ((-x)^2 = x^2)$
   d) $\forall x\, (2x > x)$
   e) $\forall n\, \exists m\, (n^2 < m)$
   f) $\exists n\, \forall m\, (n < m^2)$
   g) $\forall n\, \exists m\, (n + m = 0)$
   h) $\exists n\, \exists m\, (n^2 + m^2 = 6)$
   i) $\exists n\, \exists m\, (n + m = 4 \wedge n - m = 2)$
   j) $\forall n\, \forall m\, \exists p\, (p = (m + n)/2)$

Answer:

a) True ($x = -1$)
b) False
c) True (because for any x have $(-x)^2 = x^2$)
d) False ($x = -1$)
e) True ($m=n^2+1$)
f) True
g) True ($m=-n$)
h) False
i) True ($m=1$, $n=3$)
j) False ($m=2$, $n=3$, then $p=2.5$)

3. Translate these statements into English, where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people.

   a) $\forall x\ (C(x) \rightarrow F(x))$
   b) $\forall x\ (C(x) \wedge F(x))$
   c) $\exists x\ (C(x) \rightarrow F(x))$
   d) $\exists x\ (C(x) \wedge F(x))$

Answer:

a) Every comedian is funny
b) Every person is a comedian and is funny
c) There exists a person, if he or she is comedian, then he or she is funny
d) Some comedians are funny

4. Let Q(x, y) be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

   a) $\exists x\ \exists y\ Q(x, y)$
   b) $\exists x\ \forall y\ Q(x, y)$
   c) $\forall x\ \exists y\ Q(x, y)$
   d) $\exists y\ \forall x\ Q(x, y)$
   e) $\forall y\ \exists x\ Q(x, y)$
   f) $\forall x\ \forall y\ Q(x, y)$

Answer:

a) There are some students in your class who has sent an email message to some students in your class.
b) There are some students in your class who has sent an email message to every student in your class.
c) Every student in your class has sent an email message to some students in your class.
d) There are some students in your class who has been sent an email message by every student in your class.
e) Every student in your class has been sent an email message by some students in your class.
f) Every student in your class has sent an email message to every student in your class.

5. Translate the following statements into English where F(p) is "Printer p is out of service", B(p) is "Printer p is busy", L(j) is "Print job j is lost", Q(U) is "Print job j is queued" and U(s, r) is “Student s uses Printer r”.
   a) $\forall x\ L(x)$
   b) $\exists y\ (B(y) \wedge F(y))$
   c) $\exists c\ U(\text{‘Peter’}, c)$
   d) $\exists u\ \forall p\ (U(u, p))$
   e) $\exists y\ (U(\text{‘Ann’}, y) \wedge U(\text{‘Jo’}, y))$
   f) $\forall p\ B(p) \to \exists j\ Q(j)$
   g) $\exists j\ (Q(j) \wedge L(j)) \to \exists p\ F(p)$
   h) $(\forall p\ B(p) \wedge \forall j\ Q(j)) \to \exists j\ L(j)$
   i) $\exists y\ \forall z\ ((y \neq (\text{‘Mary’})) \wedge (U(\text{‘Mary’}, z) \to U(y, z)))$
   j) $\exists x\ \exists y\ \forall z\ ((x \neq y) \wedge (U(x, z) \leftrightarrow U(y, z)))$

Answer:

a) All the print job is lost.
b) There is a printer that is both out of service and busy.
c) There is a printer c that used by Peter.
d) There is a student who uses all the printers.
e) There is a printer that used by Ann and Jo.
f) If all the printers are busy, then there must exist some print jobs are queued.
g) If there is a print job lost and in queued, then there is a printer out of service.
h) If every printer is busy and every printer job is queued, then some job is lost.
i) There are some students except Mary who has used the printers that Mary has used.
j) There are two different students who use the same printers.

6. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a) Everybody can fool Jessica.
   c) No one can fool both Fred and Jerry.
   d) Everybody can fool somebody.
   e) There is no one who can fool everybody.
   g) No one can fool himself or herself.
   h) There is exactly one person whom everybody can fool.
   i) Nancy can fool exactly two people.

Answer:

a) $\forall x\ F(x, \text{Jessica})$
c) $\neg\exists y\ (F(y, \text{Fred}) \wedge F(y, \text{Jerry}))$
d) $\forall y\ \exists x\ F(x, y)$
e) $\neg\exists x\ \forall y\ F(x, y)$
g) $\neg\exists x\ F(x, x)$
h) $\exists y\ \forall x\ (\ (F(x, y) \wedge \forall z\ (\forall w\ F(w, z) \to (z = y))\ )$
g) $\exists x\ \exists y\ (\ F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge (y \neq z) \wedge$
$\forall z\ (\ (y \neq z) \wedge (x \neq z) \to \neg F(\text{Nancy}, z)\ )$

7. Use quantifiers and predicates with more than one variable to express these statements.
   a) There is a student in this class who can speak Cantonese.
   b) Some student in this class has visited Alaska but has not visited Hawaii.
   c) There is a student who has taken more than 21 credit hours in a semester and received all A's.
   d) A man qualifies for the marathon if his best previous time is less than 3 hours and a woman qualifies for the marathon if her best previous time is less than 3.5 hours.
   e) All students in this class have learned at least one programming language.
   f) Some student in this class grew up in the same town as exactly one other student in this class.

Answer:

a) P(x): x is a student. Q(x): x is in this class. R(x): x can speak Cantonese.
   $\exists x\ (P(x) \wedge Q(x) \wedge R(x))$
b) M(x, y): x has visited y.
   $\exists x\ (P(x) \wedge Q(x) \wedge M(x, \text{Alaska}) \wedge \neg M(x, \text{Hawaii}))$
c) P(x): x has taken more than 21 credit hours in a semester
   Q(x): x received all A's
   $\exists x\ (P(x) \wedge Q(x))$
d) P(x): x has the qualify for marathon
   Q(x): x is a man
   T(x, y): x has the chance if its best previous time is less than y yours
   $\forall x\ ((Q(x) \wedge T(x, 3.5)) \vee (\neg Q(x) \wedge T(x, 3)) \rightarrow P(x))$
e) N(x,y): x is learned a programming Language y.
   $\forall x\ \exists y\ (P(x) \wedge Q(x) \rightarrow N(x,y))$
f) G(x, y): x is grown up in the town y.
   X, y,a student
   Z town

8. Express the negations of each of these statements so that all negation symbols immediately precede predicates.
   a) $\exists z\ \forall y\ \forall x\ T(x, y, z)$
   b) $\exists x\ \exists y\ P(x, y) \wedge \forall x\ \forall y\ Q(x, y)$
   c) $\exists x\ \exists y\ (Q(x,y) \leftrightarrow Q(y,x))$
   d) $\forall y\ \exists x\ \exists z (T(x, y, z) \vee Q(x, y))$

Answer:

a) $\forall x\ \exists y \exists z\ \neg T(x, y, z)$
b) $\forall x\ \forall y\ \neg P(x, y) \vee \exists x\ \exists x\ \neg Q(x, y)$
c) $\forall x \forall y \neg (Q(x, y) \leftrightarrow Q(y, x))$
   $\forall x \forall y\ (Q(x, y)\ \leftrightarrow \neg\ Q(y, x))$
d) $\exists y\ \forall x\ \forall z\ (\neg T(x, y, z) \wedge \neg Q(x, y))$

9. Suppose that the domain of the propositional function P(x) consists of -5, -3, -1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

   a) $\exists x\, P(x)$
   b) $\forall x\, P(x)$
   c) $\forall x\, ((x \neq 1) \rightarrow P(x))$
   d) $\exists x\, ((x \geq 0) \wedge P(x))$
   e) $\exists x(\neg P(x)) \wedge \forall x\, ((x < 0) \rightarrow P(x))$

Answer:

a) $p(-5) \vee p(-3) \vee p(-1) \vee p(1) \vee p(3) \vee p(5)$

b) $p(-5) \wedge p(-3) \wedge p(-1) \wedge p(1) \wedge p(3) \wedge p(5)$

c) $\forall x\, ((x \neq 1) \rightarrow p(x))$

$\Leftrightarrow \forall x\, (\neg(x \neq 1) \vee p(x))$

$\Leftrightarrow \forall x\, ((x=1) \vee p(x))$

$\Leftrightarrow ((-5=1) \vee p(-5)) \wedge ((-3=1) \vee p(-3)) \wedge ((-1=1) \vee p(-1)) \wedge ((1=1) \vee p(1)) \wedge$
$((3=1) \vee p(3)) \wedge ((5=1) \vee p(5))$

$\Leftrightarrow (F \vee p(-5)) \wedge (F \vee p(-3)) \wedge (F \vee p(-1)) \wedge (T \vee p(1)) \wedge$
$(F \vee p(3)) \wedge (F \vee p(5))$

$\Leftrightarrow p(-5) \wedge p(-3) \wedge p(-1) \wedge T \wedge p(3) \wedge p(5)$

$\Leftrightarrow p(-5) \wedge p(-3) \wedge p(-1) \wedge p(3) \wedge p(5)$

d) $\exists x\, ((x \geq 0) \wedge p(x))$

$\Leftrightarrow ((-5 \geq 0) \wedge p(-5)) \vee ((-3 \geq 0) \wedge p(-3)) \vee ((-1 \geq 0) \wedge p(-1)) \vee ((1 \geq 0) \wedge p(1)) \vee$
$((3 \geq 0) \wedge p(3)) \vee ((5 \geq 0) \wedge p(5))$

$\Leftrightarrow (F \wedge p(-5)) \vee (F \wedge p(-3)) \vee (F \wedge p(-1)) \vee (T \wedge p(1)) \vee (T \wedge p(3)) \vee (T \wedge p(5))$

$\Leftrightarrow p(1) \vee p(3) \vee p(5)$

e) $\exists x\, (\neg p(x)) \wedge \forall x\, ((x<0) \rightarrow p(x))$

$\Leftrightarrow \exists x\, (\neg p(x)) \wedge \forall x\, (\neg(x<0) \vee p(x))$

$\Leftrightarrow \exists x\, (\neg p(x)) \wedge \forall x\, ((x \geq 0) \vee p(x))$

$\Leftrightarrow (\neg p(-5) \vee \neg p(-3) \vee \neg p(-1) \vee \neg p(1) \vee \neg p(3) \vee \neg p(5)) \wedge$
$(((-5 \geq 0) \vee p(-5)) \wedge ((-3 \geq 0) \vee p(-3)) \wedge ((-1 \geq 0) \vee p(-1)) \wedge ((1 \geq 0) \vee p(1)) \wedge$
$((3 \geq 0) \vee p(3)) \wedge ((5 \geq 0) \vee p(5)))$

$\Leftrightarrow (\neg p(-5) \vee \neg p(-3) \vee \neg p(-1) \vee \neg p(1) \vee \neg p(3) \vee \neg p(5)) \wedge (p(-1) \wedge p(-3) \wedge p(-5)$