Discrete Mathematics I

Tutorial 01 - Answer

Refer to Chapter 1.1 and 1.2

1. Let p, q, and r be the propositions

   p : You have the flu.

   q : You miss the final examination.

   r : You pass the course.

   Write these propositions using p and q and logical connectives.

   a) If you miss the final examination, you will not pass the course

   b) You have flu, or miss the final examination, or also pass the course

   Express the propositions as an English sentence.

   c) $\neg q \leftrightarrow r$

   d) $(p \wedge q) \vee (\neg q \wedge r)$

Answer:

a) $q \rightarrow \neg r$

b) $p \vee q \vee r$

c) You do not miss the final examination if and only if you pass the course

d) You have the flu and you miss the final examination or either you do not miss the final examination and pass the course.

2. Construct a truth table for each of these compound propositions.
   a) $p \to \neg q$
   b) $p \leftrightarrow q \wedge p$
   c) $p \vee q \oplus p \wedge q$

Answer:

a) $p \to \neg q$

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

b) $p \leftrightarrow q \wedge p$

p	q	$p \wedge q$	$p \leftrightarrow q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

c) $(p \vee q) \oplus (p \wedge q)$

p	q	$p \wedge q$	$p \vee q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

3. Are the following statements logically equivalent? If yes, show the proof. If no, provide a counterexample. DO NOT use truth table.
   a) $(p \wedge q)$ and p
   b) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
   c) $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$
   d) $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$
   e) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

Answer:

a) Counterexample:
When $p = T$ and $q = F$, $p \wedge q$ is false but p is T

b)

	$\neg (p \leftrightarrow q)$	
$\Leftrightarrow$	$\neg[(p \wedge q) \vee (\neg p \wedge \neg q)]$	by $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
$\Leftrightarrow$	$\neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$	by De Morgan's Laws
$\Leftrightarrow$	$(\neg p \vee \neg q) \wedge (p \vee q)$	by De Morgan's Laws
$\Leftrightarrow$	$[\neg p \wedge (p \vee q)] \vee [\neg q \wedge (p \vee q)]$	by Distributive Laws
$\Leftrightarrow$	$[(\neg p \wedge p) \vee (\neg p \wedge q)] \vee [(\neg q \wedge p) \vee (\neg q \wedge q)]$	by Distributive Laws
$\Leftrightarrow$	$[F \vee (\neg p \wedge q)] \vee [(\neg q \wedge p) \vee F]$	by Negation Laws
$\Leftrightarrow$	$(\neg p \wedge q) \vee (\neg q \wedge p)$	by Domination Laws
$\Leftrightarrow$	$p \leftrightarrow \neg q$	by $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

c)

	$(p \rightarrow q) \rightarrow r$	
$\Leftrightarrow$	$\neg(\neg p \vee q) \vee r$	by $p \rightarrow q \Leftrightarrow \neg p \vee q$
$\Leftrightarrow$	$(p \wedge \neg q) \vee r$	by De Morgan's Laws

	$p \rightarrow (q \rightarrow r)$	
$\Leftrightarrow$	$\neg p \vee (\neg q \vee r)$	by $p \rightarrow q \Leftrightarrow \neg p \vee q$
$\Leftrightarrow$	$\neg p \vee \neg q \vee r$	by Associative Laws
$\Leftrightarrow$	$\neg(p \wedge q) \vee r$	by De Morgan's Laws

Counterexample:
When $p = F$ and $q = T$, $(p \wedge \neg q) \vee r$ is unknown but $\neg(p \wedge q) \vee r$ is T

d)

	$(p \wedge q) \rightarrow r$	
$\Leftrightarrow$	$\neg(p \wedge q) \vee r$	by $p \rightarrow q \Leftrightarrow \neg p \vee q$
$\Leftrightarrow$	$\neg p \vee \neg q \vee r$	by De Morgan's Laws
$\Leftrightarrow$	$(\neg p \vee \neg q) \vee r$	

$(p \to r) \wedge (q \to r)$

$\Leftrightarrow (\neg p \vee r) \wedge (\neg q \vee r)$ by $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow [(\neg p \vee r) \wedge \neg q] \vee [(\neg p \vee r) \wedge r]$ by Distributive Laws

$\Leftrightarrow [(\neg p \wedge \neg q) \vee (r \wedge \neg q)] \vee [(\neg p \wedge r) \vee (r \wedge r)]$ by Distributive Laws

$\Leftrightarrow (\neg p \wedge \neg q) \vee (r \wedge \neg q) \vee r$ by Absorption Laws

$\Leftrightarrow (\neg p \wedge \neg q) \vee r$ by Absorption Laws

Counterexample:
When $p = F$, $q = T$ and $r = F$, $(\neg p \vee \neg q) \vee r$ is T but $(\neg p \wedge \neg q) \vee r$ is F

e) $(p \to q) \to (r \to s)$

$\Leftrightarrow (\neg p \vee q) \to (\neg r \vee s)$ by $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg(\neg p \vee q) \vee (\neg r \vee s)$ by $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (p \wedge \neg q) \vee (\neg r \vee s)$ by De Morgan's Laws

$(p \to r) \to (q \to s)$

$\Leftrightarrow (\neg p \vee r) \to (\neg q \vee s)$ by $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg(\neg p \vee r) \vee (\neg q \vee s)$ by $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (p \wedge \neg r) \vee (\neg q \vee s)$ by De Morgan's Laws

$\Leftrightarrow [p \wedge (\neg q \vee s)] \vee (\neg r \wedge (\neg q \vee s))$ by Distributive Laws

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge s) \vee (\neg r \wedge \neg q) \vee (\neg r \wedge s)$ by Distributive Laws

$\Leftrightarrow (p \wedge \neg q) \vee (\neg r \wedge s) \vee (p \wedge s) \vee (\neg r \wedge \neg q)$

Counterexample:
When left hand side does not depend on $(p \wedge s) \vee (\neg r \wedge \neg q)$

4. The proposition p NAND q is true when either p or q, or both, are false. NAND is denoted by p | q.
   a) Write down the truth table for NAND
   b) Show that p | q is logically equivalent to ¬(p ∧ q)
   c) Show that p | (q | r) and (p | q) | r are not equivalent

Answer:

a)

p	q	p | q
T	T	F
T	F	T
F	T	T
F	F	T

b)

p	q	p | q	p ∧ q	¬(p ∧ q)
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

c)

p	q	r	p | q	q | r	p | (q | r)	(p |q) | r
T	T	T	F	F	T	T
T	F	T	T	T	F	F
F	T	T	T	F	F	F
F	F	T	T	T	T	F
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	F	T	T	T	T
F	F	F	T	F	F	T

5. The following sentence is taken from the specification of a telephone system: "If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state." This specification is hard to understand because it involves two conditional statements. Find an equivalent, easier-to-understand specification that involves disjunctions and negations but not conditional statements.

Answer:

If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state.

p: directory database is opened
q: monitor is put in a closed state
r: the system is in its initial state.

$(p \land \neg r) \rightarrow q$

$(p \land \neg r) \rightarrow q$

$\Leftrightarrow (p \land \neg r) \lor q$ by $p \rightarrow q \Leftrightarrow \neg p \lor q$

$\Leftrightarrow \neg p \lor r \lor q$ by De Morgan's Laws

The directory database is not opened or
The monitor is put in a closed state or
The system is in its initial state.

6. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

Answer:

p: if Fred is not the highest paid of the three, then Janice is

q: if Janice is not the lowest paid, then Maggie is paid the most

Fred	Janice	Maggie	p	q
1	2	3	T	F
1	**3**	**2**	**T**	**T**
2	1	3	T	F
2	3	1	F	T
3	1	2	T	F
3	2	1	F	T

Fred > Maggie > Janice