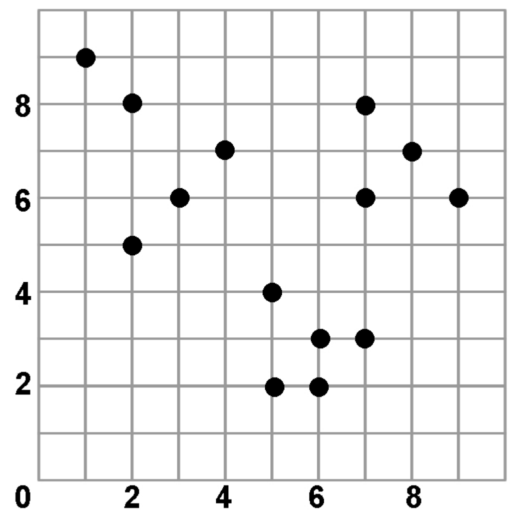
Artificial Intelligence III: Artificial Intelligence and Deep Learning

Ch06 – Unsupervised Learning

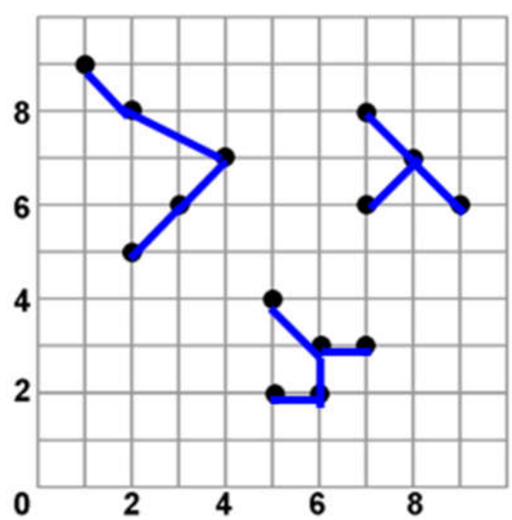
Tutorial Answer

1. Given the following dataset:



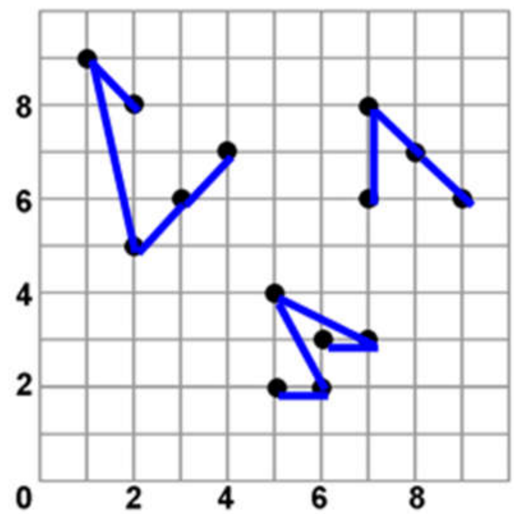
a) By using Single Linkage approach, group the samples into THREE clusters. Euclidean distance should be used.

Answer



b) By using Complete Linkage approach, group the samples into THREE clusters. Euclidean distance should be used.

Answer



c) Assume Sum-of-Squared-Error (SSE) ($S = \sum_{i=1}^{c} \sum_{x \in D_i} \|x - m_i\|^2$) as a criterion function. By setting initial points to (0, 0), (0, 10) and (8, 8), find the clusters using K-mean (k = 3).

Answer

1st Iteration:

	(0,0) 1	(0,10) 2	(8,8) 3	
(1,9)	9.0554	1.4142	7.0711	2
(2,5)	5.3852	5.3852	6.7082	1
(2,8)	8.2462	2.8284	6.0000	2
(3,6)	6.7082	5.0000	5.3852	2
(4,7)	8.0623	5.0000	4.1231	3
(5,2)	5.3852	9.4340	6.7082	1
(5,4)	6.4031	7.8102	5.0000	3
(6,2)	6.3246	10.0000	6.3246	1
(6,3)	6.7082	9.2195	5.3852	3
(7,3)	7.6158	9.8995	5.0990	3
(7,6)	9.2195	8.0623	2.2361	3
(7,8)	10.6301	7.2801	1.0000	3
(8,7)	10.6301	8.5440	1.0000	3
(9,6)	10.8167	9.8489	2.2361	3

	1	2	3
	(2,5)	(1,9)	(4,7)
	(5,2)	(2,8)	(5,4)
	(6,2)	(3,6)	(6,3)
			(7,3)
			(7,6)
			(7,8)
			(8,7)
			(8,6)
Mean	**(4.3333,3)**	**(2,7.6667)**	**(6.5,5.5)**

2nd Iteration:

	(4.3333,3) 1	**(2,7.6667) 2**	**(6.5,5.5) 3**	
(1,9)	6.8637	1.6666	6.5192	2
(2,5)	3.0732	2.6667	4.5277	2
(2,8)	5.5176	0.3333	5.1478	2
(3,6)	3.2829	1.9437	3.5355	2
(4,7)	4.0139	2.1082	2.9155	2
(5,2)	1.2019	6.4118	3.8079	1
(5,4)	1.2019	4.7376	2.1213	1
(6,2)	1.9437	6.9362	3.5355	1
(6,3)	1.6667	6.1464	2.5495	1
(7,3)	2.6667	6.8395	2.5495	3
(7,6)	4.0139	5.2705	0.7071	3
(7,8)	5.6667	5.0111	2.5495	3
(8,7)	5.4263	6.0369	2.1213	3
(8,6)	5.5478	7.1957	2.5495	3

	1	2	3
	(5,2)	(1,9)	(7,3)
	(5,4)	(2,5)	(7,6)
	(6,2)	(2,8)	(7,8)
	(6,3)	(3,6)	(8,7)
		(4,7)	(8,6)
Mean	**(5.5,2.75)**	**(2.4,7)**	**(7.4,6)**

3rd Iteration:

	(5.5,2.75)1	**(2.4,7) 2**	**(7.4,6) 3**	
(1,9)	7.7015	2.4413	8.9196	2
(2,5)	4.1608	2.0396	7.4673	2
(2,8)	6.3097	1.0770	7.6655	2
(3,6)	4.1003	1.1662	6.4000	2
(4,7)	4.5069	1.6000	5.4918	2
(5,2)	0.9014	5.6356	5.9464	1
(5,4)	1.3463	3.9699	4.8332	1
(6,2)	0.9014	6.1612	5.2498	1
(6,3)	0.5590	5.3814	4.5343	1
(7,3)	1.5207	6.0959	3.8419	1
(7,6)	3.5795	4.7074	2.4000	3
(7,8)	5.4601	4.7074	3.1241	3
(8,7)	4.9308	5.6000	1.7205	3
(8,6)	4.7762	6.6753	0.4000	3

	1	2	3
	(5,2)	(1,9)	(7,6)
	(5,4)	(2,5)	(7,8)
	(6,2)	(2,8)	(8,7)
	(6,3)	(3,6)	(8,6)
	(7,3)	(4,7)	
Mean	**(5.8,2.8)**	**(2.4,7)**	**(7.5,6.75)**

4th Iteration:

	(5.8,2.8)1	**(2.4,7) 2**	**(7.5,6.75)3**	
(1,9)	7.8409	2.4413	6.8784	2
(2,5)	4.3909	2.0396	5.7717	2
(2,8)	6.4405	1.0770	5.6403	2
(3,6)	4.2521	1.1662	4.5621	2
(4,7)	4.5695	1.6000	3.5089	2
(5,2)	1.1314	5.6356	5.3677	1
(5,4)	1.4422	3.9699	3.7165	1
(6,2)	0.8246	6.1612	4.9812	1
(6,3)	0.2828	5.3814	4.0389	1
(7,3)	1.2166	6.0959	3.7832	1
(7,6)	3.4176	4.7074	0.9014	3
(7,8)	5.3367	4.7074	1.3463	3
(8,7)	4.7413	5.6000	0.5590	3
(8,6)	4.5255	6.6753	1.6771	3

No change.

2. Given the following dataset:

Sample	Feature 1	Feature 2
s_1	1	0
s_2	2	0
s_3	3	0
s_4	5	6
s_5	6	6
s_6	7	6

a) Calculate the standardized dataset (A)

- $a = A - \mu$

Answer

$$D = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 5 & 6 \\ 6 & 6 \\ 7 & 6 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 5 & 6 \\ 6 & 6 \\ 7 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 4 & 3 \\ 4 & 3 \\ 4 & 3 \\ 4 & 3 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ -2 & -1 \\ 4 & -3 \\ 1 & 3 \\ 2 & 3 \\ 4 & 3 \end{bmatrix}$$

b) Calculate the **covariance matrix (M)** for the features in the dataset

- $Cov(D) = M = \frac{a^T a}{n}$

Answer

$$Cov(D) = M = \frac{a^T a}{n} = \begin{bmatrix} 4.66 & 6 \\ 6 & 9 \end{bmatrix}$$

c) Calculate the **eigenvalues** (λ_i) and **eigenvectors** (x_i, y_i) for the covariance matrix.

- $det(M - I\lambda) = 0$
- $M \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \lambda_i \begin{bmatrix} x_i \\ y_i \end{bmatrix}$
- Put $x_i = 1$ and convert to unit vector: $\sqrt{{x_i}^2 + {y_i}^2} = 1$

Answer

$$\det(M - I\lambda) = 0$$
$$(4.66 - \lambda) \times (9 - \lambda) - 6 \times 6 = 0$$
$$\lambda_1 = 13.213 \quad \text{or} \quad \lambda_2 = 0.454$$

$$\begin{bmatrix} 4.66 & 6 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 13.213 \begin{bmatrix} x \\ y \end{bmatrix}$$
$$4.66x + 6y = 13.213x$$
$$6x + 9y = 13.213y$$
$$y = 1.4244x$$

$$x = 1$$
$$y = 1.4244$$

$$x_1 = \frac{1}{sqrt(1 + 1.4244^2)} = 0.57$$

$$y_1 = \frac{1.4244}{sqrt(1 + 1.4244^2)} = 0.82$$

Similarly, $x_1 = 0.82$ and $y_1 = -0.57$ for $\lambda_2 = 0.454$

d) **Pick top k eigenvalues, where k = 2 (Λ_k)**

- $\Lambda_k = \begin{bmatrix} x_1 & x_2 & \dots & x_k \\ y_1 & y_2 & \dots & y_k \end{bmatrix}$, where x_i and y_i correspond to the i^{th} highest λ.

Answer

$$\Lambda_2 = \begin{bmatrix} 0.57 & 0.82 \\ 0.82 & -0.57 \end{bmatrix}$$

e) Transform the original space to the principal component space (*X*)

- $M\Lambda_k = X$

Answer

$$\begin{bmatrix} -3 & -3 \\ -2 & -1 \\ 4 & -3 \\ 1 & 3 \\ 2 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0.57 & 0.82 \\ 0.82 & -0.57 \end{bmatrix} = \begin{bmatrix} -4.18 & -0.75 \\ -3.6 & -1.07 \\ -3.03 & 4.99 \\ 3.03 & -0.89 \\ 3.6 & -0.07 \\ 4.18 & 1.57 \end{bmatrix}$$